Study of Two-Loop Neutrino Mass Generation Models

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Abstract

We study the models with the Majorana neutrino masses generated radiatively by two-loop diagrams due to the Yukawa $\rho\bar{\ell}_R^c\ell_R$ and effective $\rho^{\pm\pm}W^{\mp}W^{\mp}$ couplings along with a scalar triplet Δ, where ρ is a doubly charged singlet scalar, ℓ_R the charged lepton and W the charged gauge boson. A generic feature in these types of models is that the neutrino mass spectrum has to be a normal hierarchy. Furthermore, by using the neutrino oscillation data and comparing with the global fitting result in the literature, we find a unique neutrino mass matrix and predict the Dirac and two Majorana CP phases to be 1.40π, 1.11π and 1.47π, respectively. We also discuss the model parameters constrained by the lepton flavor violating processes and electroweak oblique parameters. In addition, we show that the rate of the neutrinoless double beta decay ($0\nu\beta\beta$) can be as large as the current experimental bound as it is dominated by the short-range contribution at tree level, whereas the traditional long-range one is negligible.

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I. INTRODUCTION

Although the data from neutrino experiments have implied that at least two neutrinos carry nonzero masses [1–5], the origin of these masses is still a mystery. Apart from the mass generation of Dirac neutrinos given by the Yukawa couplings with the existence of right-handed neutrinos (ν_R), seesaw mechanisms with type-I [6–10], type-II [11–17] and type-III [18] can generate masses for Majorana neutrinos by realizing the Weinberg operator $(\bar{L}^c_L\Phi)(\Phi^T L_L)$ at tree-level, where Φ and L_L are the doublets of Higgs and left-handed lepton fields, respectively. In these scenarios, either heavy degrees of freedom or tiny coupling constants are required in order to conceive the small neutrino masses. On the other hand, models with the Majorana neutrino masses generated at one-loop [19, 20], two-loop [21–24] and higher loop [25–28] diagrams have also been proposed without introducing ν_R. Due to the loop suppression factors, the strong bounds on the coupling constants and heavy states are relaxed, resulting in a somewhat natural explanation for the smallness of neutrino masses.

Among the loop-level mass generation mechanisms, there is a special type of the neutrino models [23, 24] in which a doubly charged singlet scalar $\rho : (1, 4)$ and a triplet $\Delta : (3, 2)$[1] under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are introduced to yield the new Yukawa coupling $\rho\bar{\ell}^c_R\ell_R$ with the charged lepton ℓ_R as well as the effective gauge coupling $\rho^{\pm\pm}W^{\mp}W^{\mp}$ due to the mixing between $\rho^{\pm\pm}$ and $\Delta^{\pm\pm}$, leading to the neutrino masses through two-loop diagrams [23]. As this model is the simplest way to realize the ρWW coupling, we name it as the minimal two-loop-neutrino model (MTM) [23]. It is interesting to note that $\rho^{\pm\pm}W^{\mp}W^{\mp}$ can also be induced from non-renomalizable high-order operators [29–31]. Although MTM can depict neutrino masses at two-loop level, the assumption on the absent of the $\bar{L}^c L\Delta$ term makes this model unnatural. To solve this problem, one can simply extend MTM by adding an extra doublet scalar, which together with Δ carries an odd charge under an Z_2 symmetry [32]. We call this model as the doublet two-loop-neutrino model (DTM). On the other hand, $\rho^{\pm\pm}W^{\mp}W^{\mp}$ could be granted by inner-loop diagrams, such as those [27, 28] with three-loop contributions to neutrino masses, in which the neutral particle in the inner-loops could be a candidate for the stable dark matter.

[1] The convention for the electroweak quantum numbers (I, Y) with $Q = I + Y/2$ is used throughout this paper.

In this study, we will demonstrate that the neutrino mass matrix can be determined in these models by the experimental data. In particular, the neutrino mass spectrum is found to be a normal hierarchy. In addition, the neutrinoless double beta decay ($0\nu\beta\beta$) is dominated by the short-range contribution at tree level due to the effective coupling of $\rho^{\pm\pm}W^{\mp}W^{\mp}$ [23, 24, 28–30, 33, 34], instead of the traditional long-range one. However, the neutrino masses in this type of the models are usually over suppressed as there is not only a two-loop suppression factor, but also a small ratio m_l/v with the charged lepton mass m_l and vacuum expectation value (VEV) $v = 246$ GeV of the Higgs field. Furthermore, the lepton flavor violation (LFV) processes could also limit the new Yukawa couplings. To have a large enough neutrino mass, the mixing angle or mass splitting between the two doubly-charged states should be large, which inevitably leads to a significant contribution to the electroweak oblique parameters, especially the T parameter. We will calculate the neutrino masses in details and check whether there is a tension between these masses and the constraint from the oblique parameter T.

This paper is organized as follows. In Sec II, we study the neutrino masses in the two-loop neutrino models. In Sec III, the constraints on the model parameters from lepton flavor violating processes and electroweak oblique parameters are studied. We present the conclusions in Sec. IV.

II. TWO-LOOP NEUTRINO MASSES

In MTM, we introduce the scalars ρ : $(1, 4)$ and $\Delta = (\Delta^{++}, \Delta^{+}, \Delta^{0})$: $(3, 2)$ under $SU(2)_L \times U(1)_Y$. The relevant terms in the Lagrangian are given by

$$\begin{aligned} -\mathcal{L} \;=\; & -\mu_\Phi^2(\Phi^\dagger\Phi) + M_\Delta^2(\Delta^\dagger\Delta) + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \bar{\lambda}_3(\Delta^\dagger\Delta)_1(\Phi^\dagger\Phi)_1 + \bar{\lambda}_4(\Delta^\dagger\Delta)_3(\Phi^\dagger\Phi)_3 \\ & + \left[Y_{ab}(\bar{L}_L^c)_a\Delta(L_L)_b + \frac{C_{ab}}{2}\rho(\bar{\ell}_R^c)_a(\ell_R)_b - \mu\Delta(\Phi^\dagger)^2 + \frac{\kappa}{2}\rho^*\Delta^2 + \bar{\lambda}\rho^*\Delta\Phi^2 + \text{H.c.}\right] \end{aligned} \tag{1}$$

where $\Phi = (\Phi^+, \Phi^0)^T$ with $\Phi^0 = (\Phi_R + i\Phi_I)/\sqrt{2}$ is the SM doublet scalar, the indices of a and b represent e, μ and τ, and the subscripts of 1 and 3 in the quartic terms stand for the SU(2) singlet and triplet scalars inside the parentheses, respectively. After the spontaneous symmetry breaking, Φ acquires a VEV of $v_\Phi = \sqrt{2}\,\langle\Phi^0\rangle$, while the neutral component of Δ also receives a VEV $v_\Delta/\sqrt{2}$, generated via the μ term. Note that by the global fitting result of $\rho_0 = 1.0000 \pm 0.0009$ [35], v_Δ is constrained to be $\lesssim 5\,\text{GeV}$, so that $v_\Phi \simeq 246\,\text{GeV}$ is a good

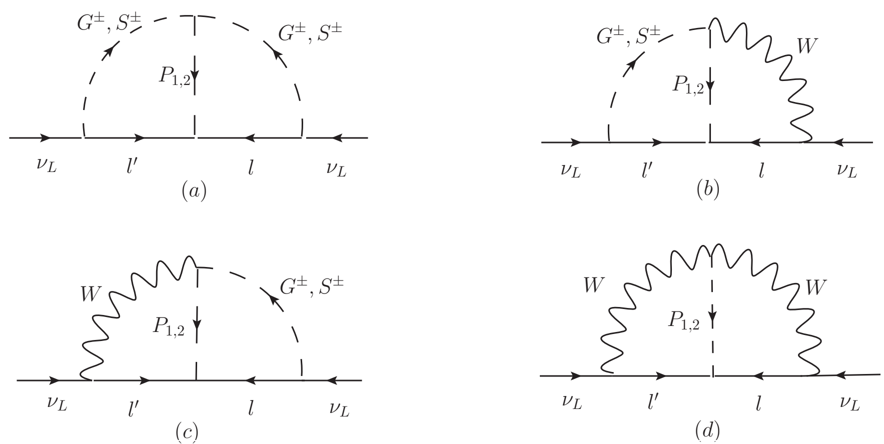

FIG. 1: Diagrams for the neutrino mass generation, where the charged states $S^{\pm}$ can be replaced by $S_1^{\pm}$ or $S_2^{\pm}$ when DTM is discussed.

approximation. The κ term in Eq. (1) can produce a mixing term between $\rho^{\pm\pm}$ and $\Delta^{\pm\pm}$, resulting in two mass eigenstates $P_{1,2}$ with masses $M_{1,2}$, respectively. We will set Y_{ab} and $\bar{\lambda}$ to be zero since they have the tree-level and logarithmic divergent two-loop contributions to neutrino masses, respectively. These two coupling can also be forbidden in a natural way by introducing a new doublet [32] or a singlet scalar [33] with an Z_2 symmetry or by replacing Δ by a higher multiplet, such as $\xi : (5, 2)$ without the discrete symmetry [36]. The scalar mass spectra of MDM are shown in Appendix A.1.

We now calculate the neutrino masses from the two-loop diagrams of Fig. 1 in the t'Hooft-Feynman gauge. The neutrino mass matrix M_ν can be written as

$$(M_\nu)_{\ell\ell'} = \frac{1}{(16\pi^2)^2} \frac{2C_{\ell\ell'} m_\ell m_{\ell'}}{v^2} \left[A_{(a)} + A_{(b)} + A_{(c)} + A_{(d)} \right] , \tag{2}$$

where the integration results $A_{(a)}$, $A_{(b)}$, $A_{(c)}$ and $A_{(d)}$, corresponding to the sub-figures (a), (b), (c), and (d) in Fig. 1, are given in Eqs. (B9)-(B11) in Appendix B, respectively. Explicitly, we find that the contribution related to $A_{(a)}$ dominates over the other three components. We note that if M_ρ is much smaller than M_Δ and the mixing angle θ between them is small, this model approximately coincides with the effective theory involving the dimension-7 operator $\rho(D_\mu\Phi)(D^\mu\Phi)\Phi\Phi$ discussed in Ref. [30].

DTM can be viewed as the extension of MTM by introducing a new doublet $\chi = (\chi^+,\, \chi^0)^T$ with $\chi^0 = (\chi_R + i\chi_I)/\sqrt{2}$. This new doublet along with Δ carries an odd charge under the Z_2 symmetry [32]. This discrete symmetry can forbid the tree-level coupling $\bar{L}^c L \Delta$ to make

the two-loop neutrino mass generation more natural. The relevant part of the Lagrangian is given by

$$
\begin{aligned}
-\mathcal{L} \;=\; & -\mu_\Phi^2(\Phi^\dagger\Phi) - \mu_\chi^2(\chi^\dagger\chi) + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \lambda_\chi(\chi^\dagger\chi)^2 + \lambda_4(\Phi^\dagger\chi)(\chi^\dagger\Phi) + M_\Delta^2(\Delta^\dagger\Delta) \\
& + \Big[\frac{C_{ab}}{2}\rho(\bar{\ell}^c_R)_a(\ell_R)_b - \mu\Delta\Phi^\dagger\chi^\dagger + \frac{\kappa}{2}\rho^*\Delta^2 + \lambda\rho^*\Delta\Phi\chi + \frac{\lambda_5}{2}(\Phi^\dagger\chi)^2 + \text{h.c.}\Big]\,. \qquad (3)
\end{aligned}
$$

Since χ does not couple to the SM fermions due to the Z_2 symmetry, the model is similar to the Type-I two-Higgs doublet model [37]. The doublet χ can also have a VEV $v_\chi/\sqrt{2} = \langle\chi^0\rangle$ due to the negative mass term of χ. We can define the mixing angle $\sin\gamma = v_\Phi/\sqrt{v_\Phi^2 + v_\chi^2}$ to characterize the scalar mixing matrix when only scalar doublets are taken into account, where $v^2 \equiv v_\Phi^2 + v_\chi^2 + 2v_\Delta^2$ with $v = 246\,\text{GeV}$. Note that the λ_5 term in Eq. (3) breaks the lepton number symmetry explicitly so that the dangerous Majaron can be avoided, while sizable values of λ_Φ and λ_χ are needed in order to give the CP even neutral scalar masses and preserve the stability of the scalar potential.

The VEV of Δ in this model is induced by the μ term in Eq. (3), which is proportional to $v_\Phi v_\chi$ with fixed values of M_Δ and μ. In the model, we have two singly-charged physical states $S_1^\pm$ and $S_2^\pm$ besides the unphysical Goldstone boson $G^\pm$, originated from the mixing among $\Phi^\pm$, $\chi^\pm$ and $\Delta^\pm$. The values of the mixing elements between the doublets and Δ are also proportional to v_Δ like MTM. Moreover, the term $\lambda\rho^*\Delta\Phi\chi$ and its hermitian conjugate provide another source for the $\rho^{\pm\pm} - \Delta^{\pm\pm}$ mixing apart from the κ term, with the contribution to θ approximately proportional to $\sin 2\gamma$. The results on the scalar mass spectra are given in Appendix A.2.

The mechanism for the neutrino mass generation in DTM is similar to that in MTM. But, the main coupling related to $\rho^{\pm\pm}$ is from the effective dimension-5 effective operator $\rho(\chi\Phi)^2$. The formula for the neutrino mass matrix is given by

$$
\begin{aligned}
(M_\nu)_{\ell\ell'} \;=\; & \frac{1}{(16\pi^2)^2}\frac{2C_{\ell\ell'}m_\ell m_{\ell'}}{v^2c_\gamma^2}\Big[\Big(\mu\frac{s_{2\theta}}{2}A_{(a1)} + \frac{\kappa}{2}A_{(a2)} + \frac{\lambda v}{2}A_{(a3)}\Big) \\
& + A_{(b)} + A_{(c)} + A_{(d)}\Big]\,, \qquad (4)
\end{aligned}
$$

where $A_{(ai)}$ and $A_{(j)}$ with $i = 1, 2$ and 3 and $j = b, c$ and d are defined in Eqs. (B12)-(B15), respectively. In Eq. (4), there is a new contribution proportional to λ, which is of $\mathcal{O}(v_\Delta/v)$. Note that the elements of the neutrino mass matrix in MTM are of $\mathcal{O}(v_\Delta^2/v^2)$.

It is crucial that the above types of the two-loop neutrino mass generation, in which $\bar{\ell}^c_R\ell_R\rho$ is the only source of the LFV, can lead to an interesting structure for the neutrino mass

matrix. The relative sizes among the matrix elements are determined by the combination factors of $C_{\ell\ell'}m_\ell m_{\ell'}$. Assuming that each value of $C_{\ell\ell'}$ is at the same order, there exist interesting hierarchies for the mass matrix elements, given by

$$(M_\nu)_{ee} \ll (M_\nu)_{e\mu} \ll (M_\nu)_{e\tau} \ll (M_\nu)_{\mu\mu} \ll (M_\nu)_{\mu\tau} \ll (M_\nu)_{\tau\tau} \, . \tag{5}$$

In particular, $(M_\nu)_{ee}$ is much less than $(M_\nu)_{\tau\tau}$ due to $m_e^2/m_\tau^2 \sim (1/\,3500^2)$. In Refs. [38–43], it has been shown that only the normal hierarchy for the neutrino mass spectrum can have the matrix textures in which $(M_\nu)_{ee}$ together with another matrix element is zero. Clearly, as the mass hierarchies in Eq. (5) naturally realize $(M_\nu)_{ee} \simeq (M_\nu)_{e\mu} \simeq 0$, both MTM and DTM predict the normal neutrino mass hierarchy.

Recall that in the standard parametrization [35, 44], the neutrino mixing matrix $V_{\rm PMNS}$ is given by

$$V_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix} , \tag{6}$$

where $s_{ij}(c_{ij})$=$\sin\theta_{ij}$ $(\cos\theta_{ij})$ with θ_{ij} being the mixing angles, δ is the Dirac phase, and α_{21} and α_{31} are the two two Majorana phases. Note that one of the Majorana phases can be absorbed by the chiral fermion fields if there exists one massless neutrino. For given values of mass square splittings and mixing angles, there are only two solutions for the three CP phases of δ, α_{21} and α_{31}, along with the lightest neutrino mass m_0, to satisfy the mass hierarchies in Eq. (5). In particular, by using the central values of the global fitting result for the normal hierarchy mass spectrum, given by [35]

$$\sin^2\theta_{12} = 0.308 \pm 0.017 \, , \; \sin^2\theta_{23} = 0.437^{+0.033}_{-0.023} \, , \; \sin^2\theta_{13} = 0.0234^{+0.0020}_{-0.0019} \, , \tag{7}$$

$$\Delta m_{21}^2 = \left(7.54^{+0.26}_{-0.22}\right) \times 10^{-5}\,\text{eV} \, , \; \Delta m_{32}^2 = (2.43 \pm 0.06) \times 10^{-3}\,\text{eV}, \tag{8}$$

we find that

$$(i) : m_0 = 5.14 \times 10^{-3}\,\text{eV} \, , \; \delta = 0.60\,\pi \, , \; \alpha_{21} = 0.11\,\pi \, , \; \alpha_{31} = 0.53\,\pi \, , \tag{9}$$

$$(ii) : m_0 = 5.14 \times 10^{-3}\,\text{eV} \, , \; \delta = 1.40\,\pi \, , \; \alpha_{21} = 1.11\,\pi \, , \; \alpha_{31} = 1.47\,\pi \, . \tag{10}$$

Note that both solutions in Eqs. (9) and (10) have the same value for m_0 but different CP phases. It is interesting to see that the predicted Dirac phase $\delta = 1.40\pi$ in (ii) of Eq. (10)

agrees well with that given by the global fitting result in Ref. [35]. Taking (ii) in Eq. (10) as the input parameters, the neutrino mass matrix is then given by

$$M_\nu = \begin{pmatrix} 0 & 0 & 1.04 e^{i1.93\pi} \\ 0 & 2.42\, e^{i0.57\pi} & 2.32\, e^{i0.50\pi} \\ 1.04\, e^{i1.93\pi} & 2.32\, e^{i0.50\pi} & 2.79\, e^{i0.55\pi} \end{pmatrix}, \tag{11}$$

in unit of 10^{-11} GeV. Note that the empty values for $(M_\nu)_{ee}$ and $(M_\nu)_{e\mu}$ can be placed by some small non-zero values when any of the parameters in (ii) is under slightly shifting.

III. CONSTRAINTS FROM LEPTON FLAVOR VIOLATION PROCESSES AND ELECTROWEAK OBLIQUE PARAMETERS

In both MTM and DTM, as the coupling matrix elements C_{ab} are the only sources of the LFV, the processes of $\ell \to \ell'\ell''\ell'''$ ($\ell \to \ell'\gamma$) with the tree-level (one-loop) contributions involving $\rho^{\pm\pm}$ could give significant constraints on C_{ab}. However, those on C_{ee} and $C_{e\mu}$ can be ignored since they do not affect the tiny matrix elements $(M_\nu)_{ee}$ and $(M_\nu)_{e\mu}$ when we discuss the neutrino mass spectrum. Among the current experimental bounds, $\mathrm{Br}(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}$ [45] is the most stringent one to limit of C_{ab}. In particular, we can obtain [28]

$$|C_{e\tau}|^2 \Big(\frac{c_\theta^2}{M_1^2} + \frac{s_\theta^2}{M_2^2} \Big) < \Big(\frac{0.336}{\mathrm{TeV}} \Big)^2 . \tag{12}$$

It is obvious that the largest allowed value of $|C_{e\tau}|_{\max}$ from Eq. (12) depends only on M_1 since s_θ is of order 10^{-2}. To account for the current experimental data on the neutrino masses as obtained in Eq. (11), the matrix element $(M_\nu)_{e\tau}$ should be around 1.04×10^{-11}GeV. As a result, we can use this value to check whether the mechanism of the neutrino mass generation can work, as shown in Fig. 2. The value of κ is taken to be $\kappa < \max(M_1, M_2)$, constrained by the perturbativity [46]. In general, a larger allowed value of κ is more possible to give a correct value of $(M_\nu)_{e\tau}$. To obtain the right values for the neutrino masses, at least one of M_1 and M_2 should roughly larger than 2.5 TeV. In Fig. 2a, $(M_\nu)_{e\tau}$ behaves approximately as an increasing function of M_2 due to the weak bound on $C_{\ell\ell'}$ from the LFV processes, while in Fig. 2b it is linearly proportional to M_2.

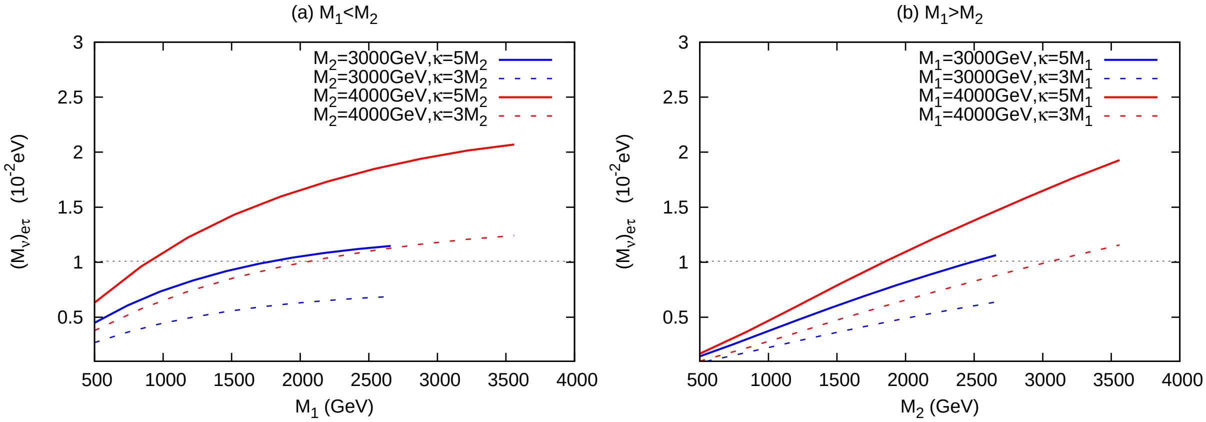

FIG. 2: Plots for $(M_\nu)_{e\tau}$ with (a) $M_1 < M_2$ and (b) $M_1 > M_2$.

The experimental constraints from the $\mu \to e$ conversion could also give some hints on M_1. To illustrate the result, we pick out some of the experimental bounds, given by $B^{\rm Au}_{\mu\to e} < 7\times 10^{-13}$ [47], $B^{S}_{\mu\to e} < 7\times 10^{-11}$ [48], $B^{\rm Ti}_{\mu\to e} < 4.3\times 10^{-12}$ [49], and $B^{\rm Pb}_{\mu\to e} < 4.6\times 10^{-11}$ [50]. For MTM and DTM, the dominant contributions come from γ and Z penguin diagrams, which lead to

$$B^{A}_{\mu\to e} = \frac{2G_F m_\mu^5}{\Gamma_A^{\rm capt}}\Big|eA_L D(A) + g_{RV}^{(p)} V^p(A) + g_{RV}^{(n)} V^n(A)\Big|^2, \tag{13}$$

where $\Gamma_A^{\rm capt}$ is the muon capture rate for the nucleus A, the coefficients A_L and $g_{RV}^{(p,n)}$ correspond to the dipole and vector contributions, and $D(A)$ and $V^{p,n}(A)$ are the overlapping functions between e and μ (see Ref. [51] for the details), respectively. Explicitly, we have

$$A_L = \frac{e\sqrt{2}}{192\pi^2 M_\rho^2 G_F}\sum_l C_{\mu l}C^*_{el}\ ,\ g_{RV(q)} = \frac{s_W^2}{2\pi^2}\frac{5M_W^2}{9M_\rho^2}Q_q\sum_l C_{\mu l}C^*_{el}, \tag{14}$$

where Q_q is the electric charge of the quark q and $g_{RV(q)}$ is the vector coupling with the quark q, mainly from the γ penguin diagram as the Z diagram is suppressed by the charged lepton masses. Based on the valence quark model, one has the relations between $g_{RV}^{(n)}$ and $g_{RV}^{(q)}$, given by $g_{RV}^{(p)} = 2g_{RV(u)} + g_{RV(d)}$ and $g_{RV}^{(n)} = 2g_{RV(d)} + g_{RV(u)}$ [51]. By taking $M_\rho = 1$TeV, $C_{e\tau} = 0.33$, and $C_{\mu\tau} = 0.0033$, we find

$$\begin{aligned} B^{\rm Au}_{\mu\to e} &= 1.4\times 10^{-14}\ , B^{\rm S}_{\mu\to e} = 8.3\times 10^{-15}\ , B^{\rm Ti}_{\mu\to e} = 1.2\times 10^{-14}\ , \\ B^{\rm Pb}_{\mu\to e} &= 9.8\times 10^{-15}\ , B^{\rm Al}_{\mu\to e} = 7.5\times 10^{-15}\ , \end{aligned} \tag{15}$$

which satisfy all the corresponding experimental limits. The improvement on the sensitivity

of the $\mu - e$ conversion [52, 53] in the future will either detect the signal or put some more stringent constraint on the models.

It is interesting to note that the neutrinoless double beta decay in our models can have a significant different feature from other models with radiative neutrino mass generations. In MTM and DTM, the short-range contributions to the decay dominate over the traditional long-range ones [23, 24], with the decay amplitudes proportional to $(M_\nu)_{ee}$. It is clear that the long-range parts can be safely neglected due to the small electron mass in $(M_\nu)_{ee}$, whereas the short-range ones are proportional only to the Yukawa coupling C_{ee}. As a result, by calculating $0\nu\beta\beta$, the upper limit on $|C_{ee}|$ could be derived, despite of the fact that it is ignored when discussing the neutrino mass matrix. The half life for $0\nu\beta\beta$ is given by [54, 55]

$$T_{1/2}^{0\nu\beta\beta} = (G_{01}|\epsilon_3^{LLL}|^2|\mathcal{M}_3|^2)^{-1} , \tag{16}$$

which leads to [29]

$$\epsilon_3^{LLL} = m_p\,(2C_{ee}^* s_{2\theta})\frac{v_\Delta}{\sqrt{2}}\frac{M_1^2 - M_2^2}{M_1^2 M_2^2} , \tag{17}$$

where G_{01} and $|\mathcal{M}_3|$ are the phase space factor and the matrix element for the hadronic sector, respectively, and ϵ_3^{LLL} is the coefficient, which is effectively related to the dimension-9 operator $(\bar{u}_L\gamma_\mu d_L)(\bar{u}_L\gamma^\mu d_L)(\bar{e}_R e_R^c)$, defined in Refs. [54, 55], and m_p is the proton mass. Note that the coefficient in Eq. (17) has no explicit dependence on the electron mass. If one takes $M_1 = 1$ and $M_2 = 1.5$ TeV in MTM, resulting in the maximal value of mixing $|\sin 2\theta| = 0.04$, the upper bounds on $|C_{ee}|$ for different target nuclei can be estimated as

$$\begin{aligned} &|C_{ee}^{(\mathrm{Ge})}| < 0.088 \ , \ |C_{ee}^{(\mathrm{Xe})}| < 0.067 \ , \ |C_{ee}^{(\mathrm{Nd})}| < 0.36 \ , \\ &|C_{ee}^{(\mathrm{Te})}| < 0.096 \ , \ |C_{ee}^{(\mathrm{Se})}| < 0.36 \ , \ |C_{ee}^{(\mathrm{Mo})}| < 0.13 \ , \end{aligned} \tag{18}$$

by comparing with experimental upper limits [56–62]. When including the effect of λ in DTM, a larger contribution to $\sin 2\theta$ could make these upper bounds on $|C_{ee}|$ to be around 20% lower.

Combing the typical value of $(M_\nu)_{e\tau}$ and the constraints from the LFV processes, at least one of M_1 and M_2 should be heavier than around 2.7 TeV in MTM. On the other hand, to get $M_1 = \mathcal{O}(10^2)$ GeV, M_2 needs to be much larger, at least 4 TeV. However, it is more difficult to get the value of M_2 less than 1 TeV with a large M_1, which means that to get $M_\rho = \mathcal{O}(100)$GeV, at least $M_\Delta \gtrsim 4$ TeV is required. Consequently, it is possible to detect the

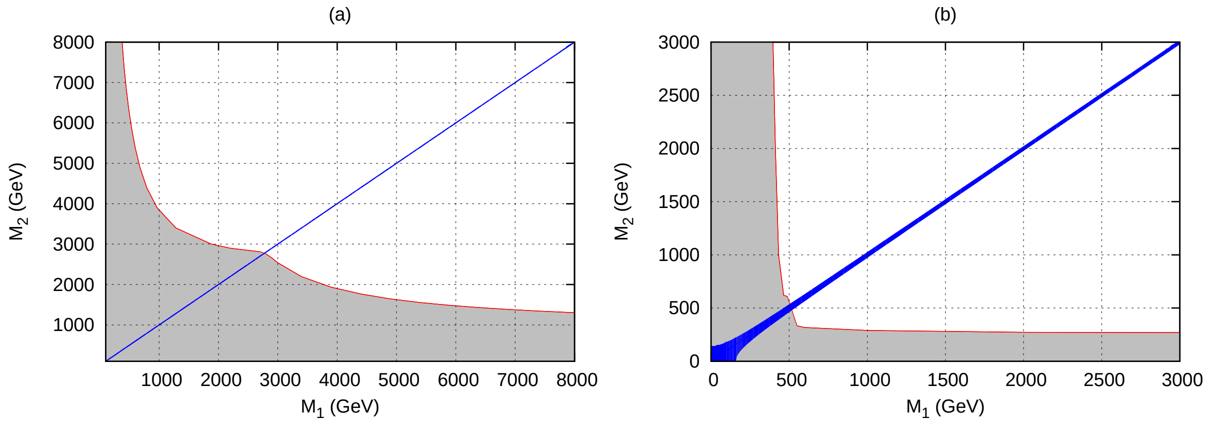

FIG. 3: Allowed regions in $M_1 - M_2$ plane for (a) MTM and (b) DTM, where $\kappa = 5\max(M_1, M_2)$ and $C_{e\tau} = (C_{e\tau})_{\max}$ are used in MTM and DTM, while $s_\gamma = 0.4$, $\lambda = 2$, $\lambda_4 = 1$ and $\lambda_5 = 1.5$ are taken in in DTM. Gray areas represent regions without enough neutrino masses, and blue regions located at $M_1 \simeq M_2$ are disallowed due to the mixing term between $\rho^{\pm\pm}$ and $\Delta^{\pm\pm}$.

signals of $\rho^{\pm\pm}$, mainly through the pair production of $\rho^{++}\rho^{--}$ and the subsequently decays with the same sign charged leptons in the final states at the LHC [30, 63]. We present the related results in Fig. 3a. Similar conclusions have been also shown in Fig. 2 and 8.8 of Refs. [33, 64], respectively, but they allowed some of the region with $M_1 \approx M_2 \approx 1.5\,\mathrm{TeV}$, which is forbidden in this paper. In DTM, the neutrino masses could be lifted up more easily by using a sizable λ as well as $\sin\theta$. Moreover, there is a new contribution to the neutrino masses from the dimension-5 operator $\rho\Phi^2\chi^2$ in DTM instead of the dimension-7 one $\rho[(D_\mu\Phi)\Phi]^2$ in MTM. As an example, if we take $s_\gamma = 0.4$, $\lambda = 2$, $\lambda_4 = 1$, and $\lambda_5 = 1.5$, along with the same values of κ and $C_{e\tau}$ in MTM, one finds that $M_2 \lesssim 400$ GeV ($\gtrsim 2$ TeV) for $M_1 \gtrsim 550$ GeV ($\lesssim 400$ GeV). The relevant result is displayed in Fig. 3b.

Finally, we briefly discuss the effects of the electroweak oblique parameters S and T in our models. First of all, in MTM and DTM, we find that the typical value of S is of order 10^{-3}, which is lower than the current experimental sensitivity. For the T parameter, the mixing between Δ and Φ gives a logarithmic divergent T due to the non-unity of ρ, but this part could be ignored when $v \ll 5\mathrm{GeV}$. In this case, the main contribution to T is given by the mixing between Δ and ρ, denoted as $T_{(\rho-\Delta)}$. It is basically a negative value whose magnitude is limited due to the small mixing angle $|s_\theta| \lesssim 0.02$. For example, taking $M_1 = 2\,\mathrm{TeV}$, $M_2 = 4\,\mathrm{TeV}$, $v_\Delta = 0.5\mathrm{GeV}$, and $\kappa = 5M_2$, it only leads to $T_{(\rho-\Delta)} = -5\times10^{-5}$.

However, in DTM the mixing angle $|s_\theta|$ can be enhanced by λ, which is independent of v_Δ. Using the input values for the above parameters, and $\lambda = 3$, we get $T_{(\rho-\Delta)} = -0.001$. Meanwhile, the mixing between Φ and χ can also provide a sizable value to the corresponding parameter $T_{(\Phi-\chi)}$. For example, $T_{(\Phi-\chi)} = 0.04$ with $s_\alpha = s_\gamma = 0.4$, $\lambda_4 = 1$, and $\lambda_5 = 1.5$, which still fulfills the experimental bound $-0.02 < \Delta T < 0.12$ at $1.5 - 1.8\,\sigma$ confident level [35]. The relevant formulae for T are summarized in Appendix C.

IV. CONCLUSIONS

We have studied the two models of MTM and DTM, in which Majorana neutrino masses are generated radiatively by two-loop diagrams due to the Yukawa $\rho\bar{\ell}^c_R\ell_R$ and effective $\rho^{\pm\pm}W^{\mp}W^{\mp}$ couplings. We have shown that the lepton violating processes, in particular, $\mu^+ \to e^+\gamma$ can give stringent constraints on the new Yukawa coupling $C_{\ell\ell'}$. By combing with the perturbativity condition for the coupling κ, the light neutrino mass element $(M_\nu)_{e\tau}$ can limit the allowed values of M_1 and M_2. In particular, we have found that only M_1 and M_2 with TeV scale can lead to the correct sizes of the neutrino masses. We have illustrated that the normal neutrino mass hierarchy is a generic feature in these two-loop neutrino mass generation models. Moreover, by using the central values of the neutrino oscillation data and comparing with the global fitting result in the literature [35], we have obtained the unique neutrino mass matrix in Eq. (11) and predicted the Dirac and two Majorana CP phases to be 1.40π, 1.11π and 1.47π, respectively. Finally, we emphasize that the neutrinoless double beta decays can be very large as they are dominated by the short-distance contributions at tree-level, which can be tested in the future experiments and used to constrain the element of C_{ee}.

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Appendix A: Scalar mass spectra in MTM and DTM

1. MTM

The non-self-Hermitian terms in Eq. (1) can be expanded as fellows:

$$\Delta(\Phi^\dagger)^2 \;=\; \Delta^{++}(\Phi^+)^{*2} + 2\frac{\Delta^+}{\sqrt{2}}(\Phi^+)^*(\Phi^0)^* + \Delta^0(\Phi^0)^{*2} \,, \tag{A1}$$

$$\rho^*\Delta^2 \;=\; 2\rho^*\left[\Delta^{++}\Delta^0 - \frac{1}{2}(\Delta^+)^2\right] . \tag{A2}$$

After obtaining the explicit forms of the scalar potential, we can write down its tadpole conditions

$$-\mu_\Phi^2 v_\Phi + \lambda_\Phi v_\Phi^3 - \sqrt{2}\mu v_\Phi v_\Delta = 0 \,, \tag{A3}$$

$$M_\Delta^2 v_\Delta - \mu\frac{v_\Phi^2}{\sqrt{2}} = 0 \,, \tag{A4}$$

which give

$$v_\Delta = \frac{\mu}{\sqrt{2}}\frac{v_\Phi^2}{M_\Delta^2} \,. \tag{A5}$$

The mixing matrices of CP odd neutral and singly charged states are written as

$$M_I^2 = \begin{pmatrix} t_\beta & -t_\beta \\ -t_\beta & 1 \end{pmatrix} M_\Delta^2 \,, \quad M_\pm^2 = \begin{pmatrix} t_\beta'^2 & -t_\beta' \\ -t_\beta' & 1 \end{pmatrix} M_\Delta^2 \,, \tag{A6}$$

where $t_\beta = 2v_\Delta/v_\Phi$ and $t'_\beta = \sqrt{2}v_\Delta/v_\Phi$. The masses of the neutral CP odd state A and singly charged states $S^\pm$ are given by $M_A = M_\Delta/c_\beta^2$ and $M_S = M_\Delta/{c'_\beta}^2$, respectively.

The doubly-charge mixing matrix is given by

$$\begin{pmatrix} M_\rho^2 & M_{12}^{\pm\pm} \\ M_{12}^{\pm\pm} & M_\Delta^2 \end{pmatrix} \,, \tag{A7}$$

where

$$M_{12}^{\pm\pm} = \frac{\kappa v_\Delta}{\sqrt{2}} \,. \tag{A8}$$

One can easily diagonalize Eq. (A7), leading to the mixing angle θ

$$t_{2\theta} = \frac{2M_{12}^{\pm\pm}}{M_\rho^2 - M_\Delta^2} \,, \tag{A9}$$

and eigenvalues of the eigenstates $P_{1,2}$

$$\begin{aligned} M_1^2 \;&=\; M_\rho^2 - \frac{s_\theta^2}{1-2s_\theta^2}(M_\Delta^2 - M_\rho^2) \,, \\ M_2^2 \;&=\; M_\Delta^2 + \frac{s_\theta^2}{1-2s_\theta^2}(M_\Delta^2 - M_\rho^2) \,. \end{aligned} \tag{A10}$$

2. DTM

The related operators with χ in Eq. (3) can be written as

$$\begin{aligned} \Delta\Phi^\dagger\chi^\dagger &= \Delta^{++}\Phi^{+*}\chi^{+*} + \frac{\Delta^+}{\sqrt{2}}(\Phi^{+*}\chi^{0*} + \Phi^{0*}\chi^{+*}) + \Delta^0\Phi^{0*}\chi^{0*} , \\ \rho^*\Delta\Phi\chi &= \rho^{--}\Big[\Delta^{++}\Phi^0\chi^0 - \frac{\Delta^+}{\sqrt{2}}(\Phi^+\chi^0 + \chi^+\Phi^0) + \Delta^0\Phi^+\chi^+ \Big] . \end{aligned} \tag{A11}$$

The minimization conditions are given by

$$-\mu_\Phi^2 v_\Phi + \lambda_\Phi v_\Phi^3 - \frac{1}{2}(\lambda_4+\lambda_5)v_\Phi v_\chi^2 - \frac{\mu}{\sqrt{2}}v_\Delta v_\chi = 0 , \tag{A12}$$

$$-\mu_\chi^2 v_\chi + \lambda_\chi v_\chi^3 - \frac{1}{2}(\lambda_4+\lambda_5)v_\chi v_\Phi^2 - \frac{\mu}{\sqrt{2}}v_\Delta v_\Phi = 0 , \tag{A13}$$

$$M_\Delta^2 v_\Delta - \mu\frac{v_\Phi v_\chi}{\sqrt{2}} = 0 . \tag{A14}$$

It is convenient to define $\bar{v} = \sqrt{v_\Phi^2 + v_\chi^2}$, and $s_\gamma = v_\chi/\bar{v}$. The singly charged and CP odd neutral mass matrices are both 3×3. In the diagonalization, we use the relation $s_\gamma \gg s'_\beta$. The transformation matrices, V_I and $V_\pm$, of CP odd neutral and singly charged states can be presented by a set of small quantities ϵ_{ij} and ϵ'_{ij}, given as

$$V_I = \begin{pmatrix} c_\gamma & -s_\gamma & \epsilon_{13} \\ s_\gamma & c_\gamma & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & 1 \end{pmatrix} , V_\pm = \begin{pmatrix} c_\gamma & -s_\gamma & \epsilon'_{13} \\ s_\gamma & c_\gamma & \epsilon'_{23} \\ \epsilon'_{31} & \epsilon'_{32} & 1 \end{pmatrix} , \tag{A15}$$

where

$$\epsilon_{32} = \frac{M_\Delta^2}{M_\Delta^2 - \lambda_5\bar{v}^2}\frac{c_{2\gamma}}{s_{2\gamma}}t_\beta , \tag{A16}$$

$$\epsilon_{31} = t_\beta , \tag{A17}$$

$$\epsilon_{13} = (-c_\gamma + \frac{c_{2\gamma}}{2c_\gamma}\frac{M_\Delta^2}{M_\Delta^2 - \lambda_5\bar{v}^2})t_\beta , \tag{A18}$$

$$\epsilon_{23} = (-s_\gamma - \frac{c_{2\gamma}}{2s_\gamma}\frac{M_\Delta^2}{M_\Delta^2 - \lambda_5\bar{v}^2})t_\beta , \tag{A19}$$

and

$$\epsilon'_{32} = \frac{M_\Delta^2}{M_\Delta^2 - \frac{1}{2}(\lambda_4+\lambda_5)\bar{v}^2}\frac{c_{2\gamma}}{s_{2\gamma}}t'_\beta , \tag{A20}$$

$$\epsilon'_{31} = t'_\beta , \tag{A21}$$

$$\epsilon'_{13} = \Big(- c_\gamma + \frac{c_{2\gamma}}{2c_\gamma}\frac{M_\Delta^2}{M_\Delta^2 - \frac{1}{2}(\lambda_4+\lambda_5)\bar{v}^2}\Big)t'_\beta , \tag{A22}$$

$$\epsilon'_{23} = \Big(- s_\gamma - \frac{c_{2\gamma}}{2s_\gamma}\frac{M_\Delta^2}{M_\Delta^2 - \frac{1}{2}(\lambda_4+\lambda_5)\bar{v}^2}\Big)t'_\beta . \tag{A23}$$

The mass eigenvalues for two CP odd states are $M_{A_1}^2 = \lambda_5 \bar{v}^2$ and $M_{A_2}^2 = M_\Delta^2$, while those for singly charged states $M_{S_1}^2 = \frac{1}{2}(\lambda_4 + \lambda_5)\bar{v}^2$ and $M_{S_2}^2 = M_\Delta^2$. Finally, the mixing angle between the doubly-charged states is given by

$$t_{2\theta} = \frac{\sqrt{2}\kappa v_\Delta + \lambda c_\gamma s_\gamma v_\Phi^2}{M_\rho^2 - M_\Delta^2} \, . \tag{A24}$$

For the mixing of CP even neutral states, one can focus on the 2×2 mixing matrix between Φ and χ, given by

$$\begin{pmatrix} 2\lambda_\Phi v_\Phi^2 & -(\lambda_4 + \lambda_5) v_\Phi v_\chi \\ -(\lambda_4 + \lambda_5) v_\Phi v_\chi & 2\lambda_\chi v_\chi^2 \end{pmatrix} . \tag{A25}$$

Rotating the states $\Phi_R = c_\alpha h - s_\alpha H$ and $\chi_R = s_\alpha h + c_\alpha H$, we obtain two mass eigenvalues M_h and M_H to be

$$M_h^2 = (2\lambda_\Phi - (\lambda_4 + \lambda_5) t_\gamma t_\alpha) v_\Phi^2 c_\beta'^2 c_\gamma^2 \, , \tag{A26}$$

$$M_H^2 = (2\lambda_\Phi + (\lambda_4 + \lambda_5) \frac{t_\gamma}{t_\alpha}) v_\Phi^2 c_\beta'^2 c_\gamma^2 \, . \tag{A27}$$

If one takes $m_h = 125.7$GeV [35] as the state found at the LHC [65, 66], M_H can be obtained as a function of $\lambda_4 + \lambda_5$ and α.

Appendix B: Two-Loop Neutrino mass

1. MTM

We calculate the neutrino masses by using the t'Hooft-Feynman Gauge. The relevant vertices corresponding to the top vertex in Fig. 1a are $-\mu \Delta^{--} (\Phi^-)^{*2}$ and $-\frac{\kappa}{2} \rho^{++*} (\Delta^+)^2$, which yield

$$\begin{aligned} - i\mathcal{M}_{\ell\ell'}^{(a)} \; = \; & \frac{i v_\Delta}{(16\pi^2)^2} \Big(\frac{C_{\ell\ell'} m_\ell m_{\ell'}}{v^2 c_{\beta'}^2} \Big) \Big[-4 \Big(\frac{\kappa v_\Delta}{v^2} \Big) \frac{M_0^2}{M_1^2 - M_2^2} (I_{111} - I_{112}) \\ & -4 \frac{\kappa v_\Delta}{v^2} \Big(c_\theta^2 (I_{111} - 2I_{121} + I_{221}) + s_\theta^2 (I_{112} - 2I_{212} + I_{222}) \Big) \Big] (\bar{\nu}_L^c \nu_L) \, , \end{aligned} \tag{B1}$$

with

$$I_{ijn} = \int_0^1 dy_2 \int_0^{1-y_1} dy_1 \int_0^1 dx_2 \int_0^{1-x_2} dx_1 \frac{2}{1-x_1} \log(y_1 [x_1 M_n^2 + x_2 M_i'^{\,2}] + y_2 x_1 (1-x_1) M_j'^{\,2}), \tag{B2}$$

where we can define $M_1' = M_W$ and $M_2' = M_S$. For Fig. 1b, we get

$$-\,i\mathcal{M}^{(b)}_{\ell\ell'} \;=\; \frac{i}{(16\pi^2)^2}\Big(\frac{\sqrt{2}C_{\ell\ell'}m_\ell m_{\ell'}}{v_\Phi}\Big)\frac{\sqrt{2}g_W^2}{2}\frac{s_{2\theta}}{2}\frac{s_{2\beta'}}{2}(M_1^2-M_2^2)(I_W^{(b)}-I_S^{(b)})(\bar{\nu}_L^c\nu_L)\;, \tag{B3}$$

with

$$I_i^{(b)} \;=\; \int_0^1 dy_2\int_0^{1-y_1}dy_1\int_0^1 dx_3\int_0^{1-x_3}dx_2\int_0^{1-x_2-x_3}dx_1\frac{-2y_1(2-x_1-x_2)}{(x_1+x_2)(1-x_1-x_2)m_i^2}\;, \tag{B4}$$

where

$$m_i^2 \;=\; y_1(x_1M_1^2+x_2M_2^2+x_3M_W^2)+y_2(x_1+x_2)(1-x_1-x_2)M_i'^{\,2}\;. \tag{B5}$$

The amplitude $\mathcal{M}^{(c)}_{ll'}$ should be equal to $\mathcal{M}^{(b)}_{ll'}$. For Fig. 1d, we have

$$-\,i\mathcal{M}^{(d)}_{\ell\ell'} \;=\; \frac{-i}{(16\pi^2)^2}\frac{2g_W^4v_\Delta}{4\sqrt{2}}s_{2\theta}(m_\ell m_{\ell'}C_{\ell\ell'})[I^{(d)}(M_{P1}^2)-I^{(d)}(M_{P2}^2)](\bar{\nu}_L^c\nu_L)\;, \tag{B6}$$

$$I^{(d)} \;=\; \int_0^1 dy_2\int_0^{1-y_1}dy_1\int_0^1 dx_2\int_0^{1-x_2}dx_1\frac{-4}{m_{(d)}^2}\;, \tag{B7}$$

where

$$m_{(d)}^2 = y_1(x_1M_i^2+x_2M_W^2)+y_2x_1(1-x_1)M_W^2\;. \tag{B8}$$

In summary, $A_{(a)}$, $A_{(b)}$, $A_{(c)}$, and $A_{(d)}$ in Eq. (2) are listed as follows:

$$\begin{aligned}
A_{(a)} \;&=\; s_{\beta'}^2\kappa\frac{M_\Delta^2}{M_1^2-M_2^2}\Big[(I_{111}-I_{112})+\frac{s_\beta'^2}{c_\beta'^2}(I_{121}-I_{122}+I_{211}-I_{212}) \\
&\quad \frac{s_\beta'^4}{c_\beta'^4}(I_{221}-I_{222})\Big]+s_{\beta'}^2\kappa\Big[c_\theta^2(I_{111}-I_{121}-I_{211}+I_{221}) \\
&\quad +s_\theta^2(I_{112}-I_{122}-I_{212}+I_{222})\Big]\;, \qquad \text{(B9)}\\
A_{(b)} \;&=\; A_{(c)} = -2s_{\beta'}^2\kappa M_W^2(I_W^{(b)}-I_S^{(b)})\;, \qquad \text{(B10)}\\
A_{(d)} \;&=\; 2s_{\beta'}^2\kappa\frac{M_W^4}{M_1^2-M_2^2}I^{(d)}\;. \qquad \text{(B11)}
\end{aligned}$$

2. DTM

If we expand the amplitudes $A_{(a1)}$, $A_{(a2)}$, and $A_{(a3)}$ up to $\mathcal{O}(v_\Delta/v)$, then the results are given by

$$
\begin{aligned}
A_{(a1)} &= 0\ ,\ A_{(a2)} = 0\ , && \text{(B12)}\\
A_{(a3)} &= s_\gamma c_\theta^2 \Big[2c_\gamma^3 \epsilon'_{31} I_{111} + 2c_\gamma s_\gamma^2 \epsilon'_{31}(I_{121}+I_{211}) + 2c_\gamma^2 \epsilon'_{13}(I_{131}+I_{311}) - 2\epsilon'_{32} s_\gamma^3 I_{221} \\
&\quad +2s_\gamma^2 \epsilon'_{13}(I_{231}+I_{321})\Big] + c_\gamma c_\theta^2 \Big[2c_\gamma^2 s_\gamma \epsilon'_{31} I_{111} - 2c_\gamma s_\gamma(\epsilon'_{31} c_\gamma + \epsilon'_{32} s_\gamma)(I_{121}+I_{211}) \\
&\quad +2c_\gamma s_\gamma \epsilon'_{13}(I_{131}+I_{311}) + 2c_\gamma s_\gamma^2 \epsilon'_{32} I_{221} - 2c_\gamma s_\gamma \epsilon'_{13}(I_{231}+I_{321})\Big] \\
&\quad -s'_\beta c_\theta^2 \Big[2c_\gamma^3 s_\gamma I_{111} - 2c_\gamma s_\gamma(c_\gamma^2 - s_\gamma^2)(I_{121}+I_{211}) - 2c_\gamma s_\gamma^3 I_{221}\Big]\ , && \text{(B13)}\\
A_{(b)} &= A_{(c)} = -2s_{2\theta}\frac{M_1^2 - M_2^2}{v} M_W^2 (\epsilon'_{31} I_W^{(b)} - s_\gamma \epsilon'_{32} I_{S_1}^{(b)} + \epsilon'_{13} I_{S_2}^{(b)})\ , && \text{(B14)}\\
A_{(d)} &= 2s_{\beta'}\frac{s_{2\theta}}{v} M_W^4 I^{(d)}\ . && \text{(B15)}
\end{aligned}
$$

where we have used the same notations as those in Appendix B1 except $M'_2 = M_{S_1}$ and $M'_3 = M_{S_2}$.

Appendix C: T parameters in MTM and DTM

The T parameter due to the mixing between ρ and Δ in MTM is given by

$$
\begin{aligned}
T_{(\rho-\Delta)} &= \frac{1}{4\pi s_W^2 M_W^2}\Big[s_\theta^2 F(M_1^2, M_{S^\pm}^2) + c_\theta^2 F(M_2^2, M_{S^\pm}^2) - F(M_\Delta^2, M_{S^\pm}^2) \\
&\quad -2c_\theta^2 s_\theta^2 F(M_1^2, M_2^2)\Big]\ . && \text{(C1)}
\end{aligned}
$$

The above result in Eq. (C1) can also be applied to DTM with replacing $S^\pm$ by $S_2^\pm$. In DTM, since the contribution from the mixing between Φ and χ should also be considered, we find

$$
\begin{aligned}
T_{(\Phi-\chi)} &= \frac{1}{16\pi s_W^2 M_W^2}\Big[(c_{\alpha-\gamma}^2 - 1)(G(M_h^2, M_W^2) - G(M_h^2, M_Z^2)) + s_{\alpha-\gamma}^2 (G(M_H^2, M_W^2) - G(M_H^2, M_Z^2)) \\
&\quad +s_{\alpha-\gamma}^2 (F(M_h^2, M_{S_1}^2) - F(M_h^2, M_{A_1}^2)) + c_{\alpha-\gamma}^2 (F(M_H^2, M_{S_1}^2) - F(M_H^2, M_{A_1}^2)) \\
&\quad +F(M_{S_1}^2, M_{A_1}^2)\Big]\ , && \text{(C2)}
\end{aligned}
$$

where the functions F, K, and G are defined by

$$G(x,y) = F(x,y) + 4yK(x,y) \ , \tag{C3}$$

$$F(x,y) = \frac{x+y}{2} - \frac{xy}{x-y}\log\frac{x}{y} \ , \ K(x,y) = \frac{x\log x - y\log y}{x-y} \ . \tag{C4}$$

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